

# Preface

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We entered the field of voting power some seven years ago. Like most people working in the area of social choice we knew that a priori voting power was commonly measured by two indices — one proposed by Shapley and Shubik [97], the other by Banzhaf [5] — which seemed to be similar to each other. We were also aware that some phenomena associated with these indices were described in the literature as paradoxes. Wanting to learn more about them, we subjected them to closer examination and concluded that these phenomena were not too astonishing. However, in analysing the so-called paradoxes of redistribution and large size (see §§ 7.3 and 7.2, respectively), we began to suspect that the Banzhaf index might display much more extreme paradoxical behaviour. This led to the discovery in [29] of the transfer paradoxes (see § 7.8).

At a deeper level, we remained dissatisfied with the conceptual foundations of the subject. It seemed to us that the underlying notion of voting power — that real magnitude which is supposed to be measured by the indices — was insufficiently analysed and clarified. The technical results we were obtaining seemed to demand such clarification, and at the same time pointed the way to it. Then — particularly in the course of our joint work on [36] with William Zwicker (see § 7.9) — we gradually realized that there were in fact *two* quite distinct notions of voting power, which were commonly conflated with each other. While the Shapley–Shubik index was based on a notion, derived from cooperative game theory, of voting power as expected share in a fixed total prize (see § 6.1), the Banzhaf index (which, as we found out in the meantime, had been proposed much earlier by Lionel Penrose [78]) was based on a notion of voting power as probability of influencing the decision of

a voting body — a quantity whose total is not constant (see § 3.1). Yet, the Banzhaf index was inappropriately treated by most writers on voting power as a game-theoretic measure of constant-sum payoff, merely a variant of the Shapley–Shubik index.

As a matter of fact, the distinction between the two notions of voting power had been urged by Coleman [20]; but his objections went unheeded by most researchers, who continued to conflate the two underlying notions. We ourselves had shared the confusion for a while, and in [29] we mistakenly dismissed the Banzhaf index, due to our failure to arrive at a true explanation of its paradoxical behaviour.

When we started to look into a priori voting power, a prevalent view among political scientists was that this subject was somewhat old-fashioned. It may have been all the rage in the 1960s, but by the 1980s it had become an exhausted mine, where it is no longer possible to unearth anything of real value. Meantime, recent research had moved into the field of *de facto* voting power, which depends on the local institutional setting of the decision-making body. This view is still held by those who have not kept up with even more recent work on a priori voting power.

But it became increasingly evident to us that earlier work on a priori voting power had left wide gaps not only in the superstructure of the theory but at its very foundations, which were left in a confused state. It was perhaps not accidental that while this theory was more than half a century old, and the literature on it was fairly large, there was as yet no monograph devoted entirely to it. Meantime, many practitioners (like most scientists who do statistical testing) employed the mathematical formulas mechanically, unconcerned about their justification — much as a cook uses a recipe without worrying about the underlying chemistry.

Moreover, although we would be the last to deny the importance of research into actual, a posteriori, voting power in specific institutions, we believe that a robust theoretical approach to such problems must be based on a sound theory of a priori voting power. Actual voting power is, after all, a result of superposition of specific institutional factors on an a priori ground. Without a clear

understanding of the latter, there is little hope of theoretical clarity and empirical coherence in studying the composite edifice as a whole.

The main purpose of this book is therefore to attempt a systematic critical examination and exposition of the foundations and methodological presuppositions of the theory of a priori voting power.

While presenting many technical results, some of them new, we have made no attempt at comprehensive up-to-date coverage of all that is known in this field. And we have resisted the temptation to indulge in mathematical development for its own sake or for purely ornamental purposes. The technical results we do present are those we believe to have foundational significance. Almost all of them are actually used directly or indirectly in the conceptual analysis. Thus, for example, the technical exposition in §§ 3.3 and 3.4—which contains some new results, including a precise statement and proof of the second square-root rule—is required for analysing issues such as the distinction between equal suffrage and majority rule in a two-tier decision-making system, which is fundamental for representative democracy. Similarly, the novel mathematical framework outlined in Ch. 8, in which abstention is admitted as a distinct option, is in our view an essential extension of the foundation of the theory of voting power. We use it to examine critically two of the stock examples used in the literature as illustrations: the distribution of power in the US legislature and the UN Security Council.

The foundational intent of our project imposes the need for mathematical rigour. The theory of voting power is ineluctably mathematical, and it would be impossible in a book of this sort to relegate all the mathematics to technical appendixes, still less to dumb it down. This would be like Hamlet without the prince, or with the prince in a small walk-on role. Five chapters (2, 3, 6, 7 and 8) are devoted to developing the mathematical foundations of the theory, and subjecting them to critical examination.

Of course, the book is not written primarily for expert mathematicians. We hope that large parts of it will be accessible and

useful to students and researchers in the area of social choice, as well as in related areas concerned with aspects of decision-making by vote and distribution of voting power: political science, economics, business administration and constitutional law. We have tried to make the five mathematical chapters as reader-friendly as we could, in two ways. First, the mathematical knowledge that we presuppose is actually quite modest. Apart from general set-theoretic concepts, terminology and notation that are nowadays standard *lingua franca* in almost all mathematical discourse, we only presuppose a modicum of familiarity with elementary probability theory. In fact, almost all the probability spaces we consider are finite. The sole exception is at the very end of §6.3; and even there we only use a bit of elementary calculus. Second, the more technical formal definitions and theorems are accompanied by Remarks in which they are explained in plainer language; and they are often illustrated by simple examples.

The reader should note the difference between Remarks and Comments. The former contain, in addition to informal explanations of formal statements, also some supplementary technical, historical and bibliographic details. The latter, on the other hand, contain discussion of methodological and foundational matters. Thus, the ‘message’ of this book is to be found in the Comments.

A reader who wishes to omit as much of the mathematical development as possible is advised, when reading these five chapters, to concentrate on the Comments, and refer back to the mathematical statements as the need arises. To make this as easy as possible, we have supplied copious cross-references in the text, as well as a Technical Index.

Two chapters are devoted to extensive case studies. These too are not undertaken solely for their own sake, but in order to illustrate how fundamental theoretical issues turn up, and are dealt with for better or worse, in real life. Ch. 4 is devoted to a series of US court cases concerning the application of the principle of equal suffrage (‘one person, one vote’). We subject the courts’ opinions to critical examination in the light of the theory developed in the preceding chapter. In particular, the frequently cited (and misrepresented) case of weighted voting in Nassau County,

NY, is discussed in considerable detail and updated. Ch. 5 outlines the history of weighted voting (aka ‘qualified majority voting’) in the European Community’s Council of Ministers from the Community’s foundation in 1958, through its four successive enlargements, up to the time of writing (spring 1998). This weighted voting system is subjected to detailed theoretical analysis.

In view of the main purpose of this book, we could not avoid making outspoken, sometimes even harsh, critical comments on some statements and positions found in the voting-power literature that we believe to be misleading, confused or simply erroneous. In no case are these comments intended personally against the particular authors cited (who, besides, may have changed their views by now). The targets of our criticism are merely a sample: a mistake made by one author was surely shared by many others—in some cases including ourselves. In all fairness we must admit that it has taken us quite long to rid ourselves of some common confusions and misconceptions. And it wasn’t easy. Of course, in hindsight ‘all difficulties are but easy’, as the motto of our book puts it.

**Notes** (i) For the sake of brevity we often refer to a work listed in the Bibliography by means of its number, without mentioning the author(s). However, in the General Index such references are listed under the authors’ names. For example, in fn. 14 on p. 47 we refer to [27] without mentioning the authors. But the Index lists this reference under the names of both authors, Dubey and Shapley.

(ii) The Technical Index is in lieu of a glossary. It lists the locations in the text where a technical term is defined, redefined or explained, but not where it is merely used.

(iii) The US court cases cited are not included in the Bibliography but are listed separately. In quoting from opinions of the US Supreme Court, we follow the style and punctuation of the Lawyers’ Edition, but the page references are to the original edition (whose pagination is also indicated in the Lawyers’ Edition).

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